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INVERSION OF BAND MATRICES

AFOSR-77-3183

FINAL REPORT: AF GRANT [illegible] & 75-2844

Yasuhiko Ikebe, Principal Investigator

This is the summary report for the project entitled "Matrix methods for special functions arising in applications" supported through AF Grants 77-3138 & 75-2844.

An extremely simple and efficient method for computing the zeros (possibly complex) of Bessel functions $J_m(Z)$, of any real order and of their derivatives was developed and tested. For the complex zeros (the case $m < -1$) the numerical experiment shows that the method computes the zeros in the increasing order of distance from the origin. The method developed in this project is the first known systematic method for computing the complex zeros of Bessel functions of order less than -1. The method consists of writing the well-known three-term recurrence relations in matrix form, thus reformulating the problem of finding the zeros as eigenvalue problem for infinite matrices. A very complete analysis for the rate of convergence has been obtained for the zeros of Bessel functions of real order greater than -1.

It was also discovered in the course of the project that the earlier method found by this principal investigator for computing the zeros of the regular Coulomb wave functions $F_L(\eta, \rho)$ and of their $\rho$ derivatives, where L and $\eta$ are parameters ($L = o, 1, 2, \ldots, -\infty < \eta < \infty$) and $\rho > 0$, can be extended to the case where L is not necessarily a nonnegative integer. The rate of convergence has been seen to be extremely rapid. The numerical method again consists of formulating the three-term recurrence relations as the eigenvalue problem for infinite matrices.

The eigenvalue problem for Mathieu's equation $y'' + (\lambda - 2q\cos 2x)\, y = 0$ with $\pi$ or $2\pi$ periodic solutions has been shown through this project to be most tractable when formulated as eigenvalue problem for infinite matrices, which again derives from the well-known three-term recurrence relations existing among the coefficients of the trigonometric series representation of the $\pi$- or $2\pi$- periodic solutions of Mathieu's equation. The method has been shown to be applicable to the complex case (namely, q complex). The complete justification of the method via the operator theory has been obtained, where the real or complex case of q can be treated in a unified manner. The traditional approach via continued fraction equation formulation is completely powerless for the complex case.

The eigenvalue problem for Lamés equation $w'' + \{h - n(n+1)k^2 sn^2(z, k)\} w = 0$, has been approached via the matrix method by other authors. However, only numerical experiments has been reported, where error analysis including rate of convergence is completely missing. The investigation with the support of the present grant has been initiated. At the time of writing a complete justification of the matrix method under consideration has been obtained.

It is to be noted that the prerequisite for the matrix method is the existence of good matrix eigenvalue solver. This is provided by the EISPACK package.